Mysterious Discontinuity of Quantum Correlation

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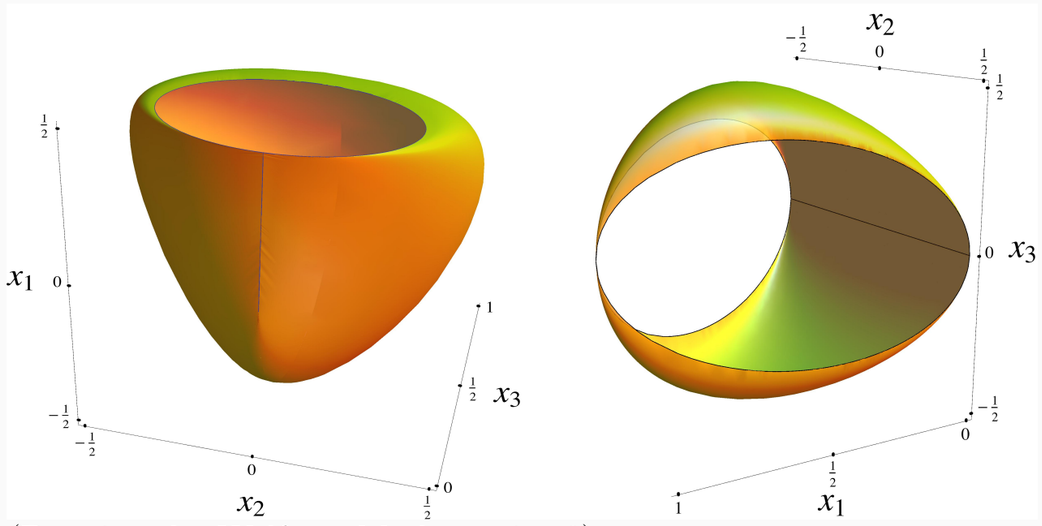
Abstract

Projections of spaces of probability distributions are polytopes. In contrast, projections of quantum state spaces have flat and curved boundary portions whose coincidence produces discontinuities in the maximum-entropy principle and derived correlation quantities.

Do the discontinuities have a physical meaning?

Projections of State Spaces

The space of qu-d-it states $\mathcal{M}_d$ is the convex set of positive semidefinite d-by-d matrices of trace one. Examples of 3D-projections of $\mathcal{M}_3$ [4]:



(Drawings by Wolfram Mathematica 9)

The convex hull of each surface is a projection of $\mathcal{M}_3$. Observe the 1D flat boundary portions!

Maximum-Entropy Inference

self-adjoint d-by-d matrices $u_1, \ldots, u_r$

linear map $\mathbb{E} : \mathcal{M}_d \to \mathbb{R}^r$, $\rho \mapsto \operatorname{tr}(\rho u_i)_{i=1}^r$ and projection $\mathbb{L} := \mathbb{E}(\mathcal{M}_d)$

MaxEnt inference map $\rho^* : \mathbb{L} \to \mathcal{M}_d$,
$\rho^*(x) = \operatorname{argmax}\{S(\rho) \mid \rho \in \mathcal{M}_d, \mathbb{E}(\rho) = x\}$,
von Neumann entropy $S(\rho) = -\operatorname{tr} \rho \log(\rho)$

ρ^* can be discontinuous [1] for $d \geq 3$, $r \geq 2$

Convex Geometry

The set-valued *face function* $F : \mathbb{L} \to \mathbb{L}$ maps $x \in \mathbb{L}$ to the union of all closed segments in $\mathbb{L}$ whose open segment contains x.

Theorem [3] (Lower semi-continuity)

Let $(x_i) \subset \mathbb{L}$ and $x \in \mathbb{L}$. If $\rho^*(x_i) \overset{i\to\infty}{\to} \rho^*(x)$ then

$$\dim F(x) \leq \liminf_{i\to\infty} \dim F(x_i). \quad (1)$$

If $\dim F(x_i) \equiv 0$ then (1) shows that $x \in \mathbb{L}$ is a discontinuity point of ρ^*, if x is a limit of extreme points but no extreme point itself(*).

(*) Conjectured in [6]

Three-Party Correlation

The *irreducible three-party correlation* [5] inherits from ρ^* a discontinuity [3] at the GHZ-state $(|000\rangle + |111\rangle)/\sqrt{2}$ while being regular in many other respects [2].

Challenges

Can we compute the discontinuities of ρ^*, or of the irreducible correlation?
Does the discontinuity of ρ^* have a meaning in the theory of topologically ordered systems [6]?

References

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